Regularities in the Temperature Dependence of the Thermal Conductivity of Refractory Products and Two Metallic Alloy Groups

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The temperature dependence of the thermal conductivity of refractory products is calculated from the linear course between the logarithms of the thermal conductivity and the logarithms of the temperature in Kelvin. Two exponents were generated, n and N where the reference point for N is the exponent to the bases of the Euler constant e (thermal conductivity at 1 K) and the temperature dependence is the exponent n to the temperature in Kelvin.
Within different refractory groups there is a linear dependence between the exponents N and n with a very high coefficient of determination. The same relation was found for two groups of metallic alloys.
High coefficients of determination within refractory groups enables to calculate the temperature-thermal conductivity values within the range from 0 up to 1400 °C from one single thermal conductivity/temperature value and gives the possibility to calculate the integral mean of the thermal conductivity between two temperatures.

1 Introduction

Refractory and insulating materials used in industrial processes, show different thermal conductivities and temperature dependence.
Knowing the course of thermal conductivity with temperature is necessary to carry out thermal calculations.
The dependency of the thermal conductivity from temperature can be measured by different methods [1]. Here all values were measured with the Klasse Method [2]. The dependency of thermal conductivity from porosity for dense material is described by Traustel, S., et al. [3] and France, J. et Kingery W. D. [4]. These equations are only valid for limited differences in the porosity.
Mathematic approximations for thermal conductivity/temperature dependence, especially 1/T approximations are described in several papers [5–9].
But these dependencies only apply to the special material, not to the material group of the sample.
In his dissertation [10] the author of this paper reports a clear regularity for the dependence of the thermal conductivity from temperature for all types of magnesia bricks.

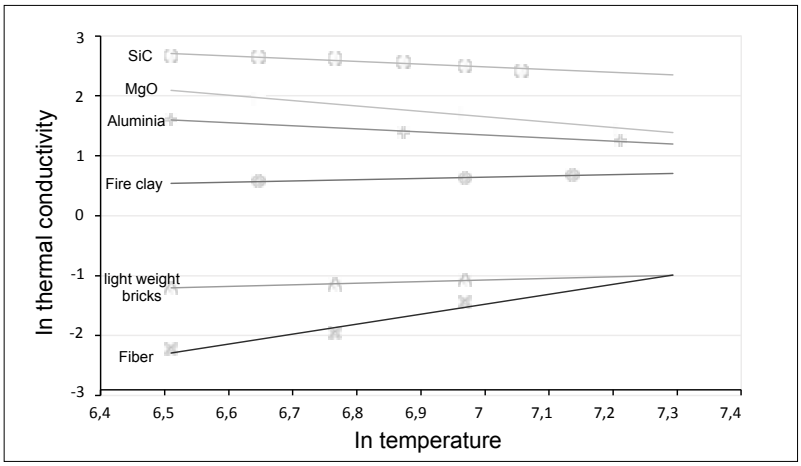

Fig. 1 Logarithm of thermal conductivity vs. logarithm of abs. temperature for some refractory materials

The aim of this paper is, using a plurality of records from refractory and insulating material, all measured by the same method, to find similar principles for the different refractory and insulating material groups and two alloy groups [11].
These principles should allow safe prediction for the course of the thermal conductivity with temperature, even with only one measured value of the thermal conductivity. As this is a function of temperature it

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Keywords: refractories, alloys, thermal conductivity

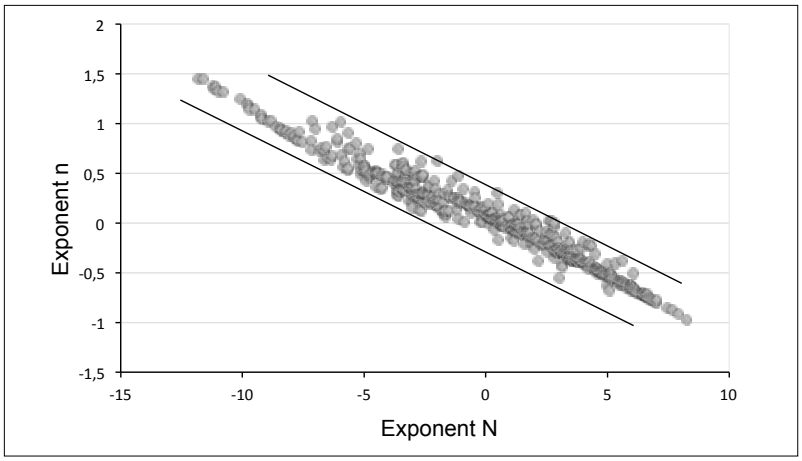

Fig. 2 Area of the calculated n/N dependences from the underlying records of thermal conductivity measurements

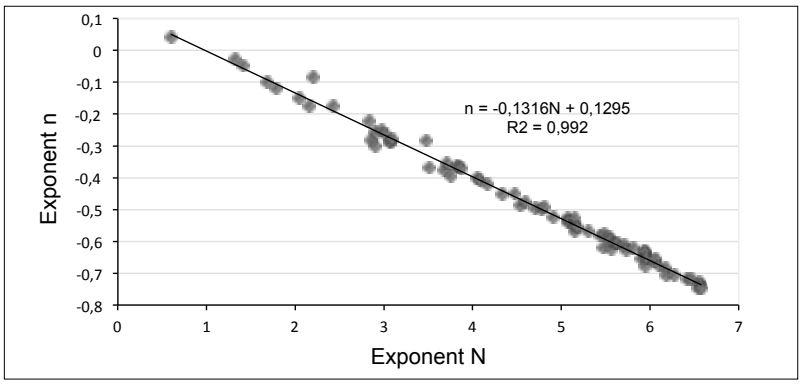

Fig. 3 n(N) dependence for magnesia bricks

is possible to calculate the integral mean between two temperatures for thermal calculations.

Such clear regularities in thermal conductivity within groups of refractory and insulating materials have not been described until now in literature.

2 Basic approach

Plotting the logarithms of the thermal conductivity λ against the logarithms of the temperature T in Kelvin results in linear dependence (equation 1) for different refractory groups (Fig. 1).

$$\ln \lambda_{eff} = n \cdot \ln\left(\frac{T}{T_0}\right) + N \qquad (1)$$

or

$$\lambda_{eff} = e^N \cdot \left(\frac{T}{T_0}\right)^n \qquad (2)$$

with

$$e^N = \lambda_o \qquad (3)$$

λ_0 = the fictitious thermal conductivity with the reference temperature T_0 = 1 K.

3 Calculation of the exponents n and N

The thermal conductivity was measured by the Klasse Method [2] in the former Didier Research Centre in Wiesbaden/DE between 1975 and 1995. Base are over 500 records. Taken into account were only records that could be clearly assigned to the different material groups and if the number of those records where more than five.

The thermal conductivity was measured for three temperatures within the temperature range from 300 °C up to 1400 °C. N and n were calculated with the λ values (λ_1 / T_1) at highest and (λ_2 / T_2) at lowest temperature.

As a control the value for the thermal conductivity between these two temperatures was calculated from N and n and compared with the measured values. Most deviations were in the range of ±3 %. The max. deviation was –12 %.

The determination of N and n from the values (λ_1 / T_1) and (λ_2 / T_2) was performed as follows [10]:

$$n = \ln\left(\frac{\lambda_1}{\lambda_2}\right) / \ln\left(\frac{T_1}{T_2}\right) \qquad (4)$$

and

$$N = \ln\left(\frac{\lambda}{T^n}\right) \qquad (5)$$

If in the investigated temperature range (from 300 °C up to 140 °C) the values of N and n are plotted against each other, a good approximation, a linear equation results in the form of

$$n = a\,N + b \qquad (6)$$

In Fig. 2 the values of all calculated exponents N and n are plotted against each other.

The apparent scattering largely disappears when n and N are calculated according to equations (4) and (5) for each refractory material group.

4 Dependence of the exponent n and N for the example magnesia bricks

Singled out as example for a variety of MgO bricks the tendency is shown in Fig. 3. The coefficient of determination (COD) is R^2 = 0,992; the scattering is therefore low. This enables to calculate from one measured value (λ^* / T^*) the entire thermal conductivity/temperature profile.

4.1 Calculation of the thermal conductivity of magnesia bricks from the exponent n and N and their temperature dependence

From equation (4) results the exponent n for the following measured values ($\ln \lambda_1$ / $\ln T_1$) = (2,0 / 6,6) and ($\ln \lambda_2$ / $\ln T_2$) = (1,3/7,3):

$$n = (\ln \lambda_1 - \ln \lambda_2) / (\ln T_1 - \ln T_2)$$
$$n = (2,0 - 1,3) / (6,6 - 7,3) = -1,0 \qquad (4a)$$

From equation (5) follows:

$$N = \ln \lambda_2 - n \cdot \ln T_2$$
$$N = (1,3 - (-1) \cdot 7,3 = 8,6 \qquad (5a)$$

From this results the equation according to equation (2):

$$\lambda_{eff} = e^{8,6} \cdot \left(\frac{T}{T_0}\right)^{-1} \quad (2a)$$

This analysis was performed for each of the illustrated 85 MgO bricks in Fig. 3, with the result that, according to equation (6) for all together yields the following equation:

$$n = -0,1316 \cdot N + 0,1295 \quad (7)$$

If only one pair of values (λ^* / T^*) for any MgO brick is known, it can be calculated an overall temperature dependence due to the validity of equation (7).
Required are the unknown exponents N and n. They will be determined with the known (λ^* / T^*) value, for instance

(λ^* / T^*) = 5,0 $\frac{W}{m \cdot K}$ / 673 K), and with equations (5a) and (7):

$$N = \ln \lambda^* - (-0,1316 \cdot N + 0,1295) \cdot \ln T^*$$
$$N = \ln 5 - (-0,1316 \cdot N + 0,1295) \cdot \ln 673 = 5,35 \quad (5b)$$

$$n = -0,1316 \cdot N + 0,1295 = -0,57 \quad (7a)$$

N and n are the same obtained already during the evaluation of a value pair in Fig. 1, which is not known, however, if you only present one measured value and Fig. 3.
Now N = 5,35 and n = –0,57 are put in Equation 2 and 3 and λ(T) can be calculated. Fig. 4 shows the thermal conductivity from 100 K (–173 °C) for this example up to 1500 K (1227 °C). Precondition for the calculation is a steady course of the values and no changes in modifications within the measured temperature range.
With known exponents n and N, equation (2) can also be used to calculate the integral mean of the thermal conductivity between two temperatures. The integral mean takes into account a nonlinear course of the temperature dependence:

$$\lambda_{eff} = e^N (T/T_o)^n \quad (2)$$

the integral mean results in

$$\bar{\lambda} = e^N / (b-a) * \int_a^b T^n \, dT = e^N / ((b-a) * (n+1)) * [T_b^{n+1} - T_a^{n+1}] \quad (8)$$

For the example in Fig. 4 with N = 5,35 and n = –0,57 the integral mean between 300–1300 K results in 5,0 W/m·K:

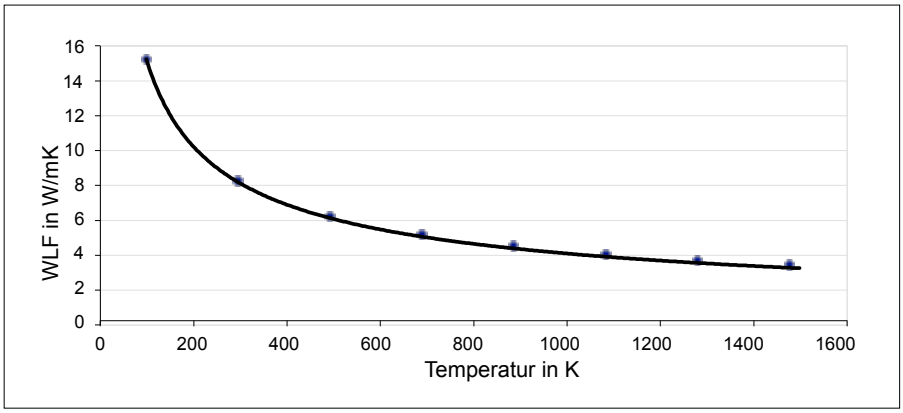

Fig. 4 Calculated thermal conductivity vs. temperature for N = 5,35 and n = –0,57

The arithmetic mean is 17 % higher than the integral mean.
The higher the temperature dependency, the more accurate is the value of the integral mean instead of arithmetic mean.

5 Results for the n/N dependence for different refractory raterial groups from the records and some metallic alloys from literature [11]

In Tab. 1 the n/N dependence for different refractory groups with more than 5 records is summarized. Values for the coefficient of determination higher than 0,985 are sufficient to calculate a temperature dependence within the refractory material group. If it is lower within a material group, the materials in the group have to be differentiate more.
Examples where more differentiate is required are for instance magnesia bricks with different additions, like baddeleyit, or chrome ore in burned magnesia bricks or differences in bonding, between burned bricks and carbon-bonded bricks with different amounts of C or graphite additions.
In this cases it is necessary to calculate N/n dependence separately for the different types of material.
Differences in N/n dependence are also between silica bricks, and bricks based on fused silica or translucent vitreous silica. In Tab. 1 this was not taken into account with the result that the coefficient of determination comes below 0,89. There were not enough samples for the determination of the thermal conductivity for each type of material, but the course shows the same tendency as all other materials.
Fireclay bricks, fireclay monolithic and SiC-bricks can have different bonding systems and different amount of the main components. As this is not taken in account, the coefficient of determination is too low for calculate a complete temperature dependence.
Unexpected but very interesting are the dependence for metallic alloys, which show for Cu, Al, Mg containing and non-containing alloys the same dependence like refractories.
Metals conduct with photons, and the Wiedemann-Franz law is valid, refractories conduct with phonons and radiation. There is, till now, no explanation why both materials can be calculated in the same way.

6 N/n dependence for some material groups

Fig. 5 shows the n/N dependence for some refractory material groups.
There are clear differences between materials with a high porosity and materials with predominant ionic bonding and metallic bonding.

6.1. Magnesia bricks

MgO-grades with N-values less than 1,5 do have more than 40 % chrome ore. The dependence with temperature is low. Above N = 1,5 there are first of all chemical bonded, water free temperature treated bricks. With rising values of the exponent N the bricks become denser and show higher crystallized texture with raising temperature/thermal conductivity dependence.
There is no dependence from MgO-content to be seen. For all grades the contacts within the texture, the growth of the crystals and the porosity have big influence on the thermal conductivity.

Tab. 1 Dependence (n/N) for different refractory and two alloy material groups

Group of Material	Number of Records	Equation	Coefficient of Determination	Range of Exponent N
Magnesia Bricks	85	n = –0,13165N + 0,1295	0,9920	0,50 … 6,50
Mag-Graphite Bricks	11	n = –0,13170N + 0,2548	0,9842	1,50 … 6,00
Corundum Bricks	73	n = –0,12190N + 0,0880	0,9856	–1,00 … 5,00
Bauxite Bricks	9	n = –0,13160N + 0,0946	0,9815	–1,00 … 4,00
Mullite Bricks	7	n = –0,13860N + 0,0771	0,9979	–1,50 … 3,70
Fireclay Bricks	17	n = –0,12440N + 0,0800	0,9753	–2,50 … 0,25
Fireclay Monolithic	32	n = –0,13130N + 0,0215	0,9365	–4,00 … –0,50
Silica Bricks	14	n = –0,10890N + 0,1829	0,8897	3,70 … 2,60
Zircon-Mullite Bricks	8	n= –0,14160N + 0,1140	0,9779	–0,75 … 1,75
Heat Insulating Material	114	n = –0,12470N – 0,0618	0,9821	–12,00 … 3,00
Heat Insulating Bricks	82	n = –0,12132N – 0,0465	0,9711	–8,00 … 3,00
Heat Insulating Monolithic	9	n = –0,13310N – 0,1270	0,9913	–5,00 … –1,80
Fibre Material	23	n = –0,15090N – 0,3148	0,9968	–12,00 … –2,80
SiC-Bricks	11	n = –0,12450N + 0,3036	0,9547	2,00 … 5,50
Carbon Bricks	10	n = –0,12180N + 0,3556	0,9960	3,20 … 7,00
System Al_2O_3–SiO_2	120	n = –0,13160N + 0,0885	0,9927	–3,80 … 5,00
System SiC - C	19	n = –0,12470N + 0,3255	0,9953	–3,20 … 7,00
Alloys with Cu, Al, Mg	13	n = –0,13200N + 0,6984	0,9943	–4,00 … 7,00
Alloys without Cu, Al, Mg	52	n = –0,13300N + 0,4599	0,9988	–6,00 … 8,50

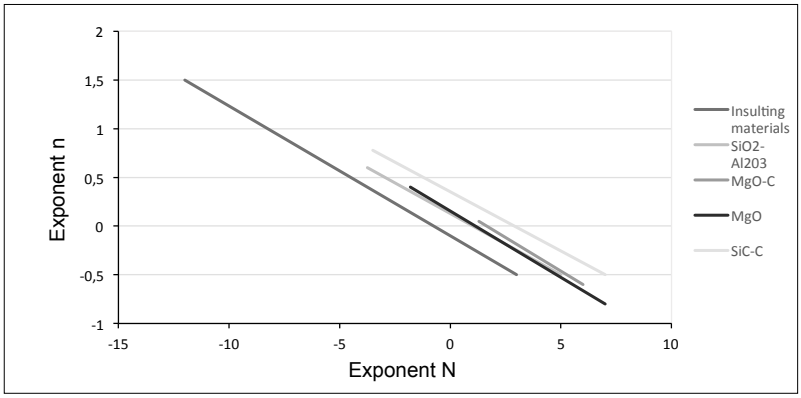

Fig. 5 Dependence n(N) for some refractory material groups

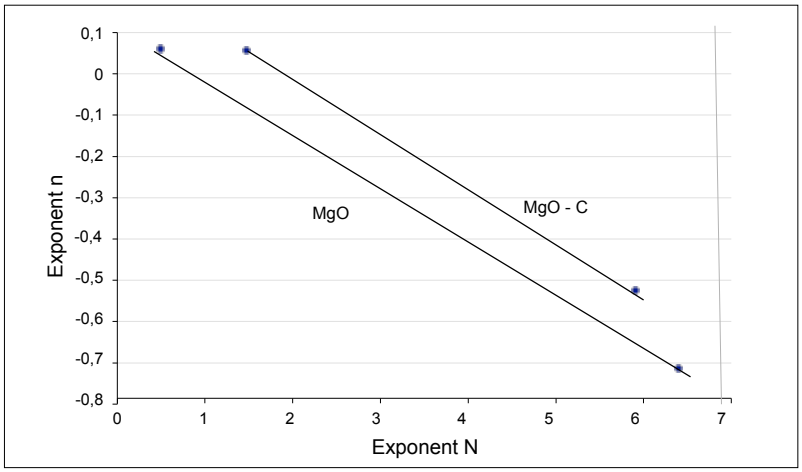

Fig. 6 Dependence n(N) for magnesia bricks and graphite containing carbon bonded magnesia bricks

Carbon bonding and graphite contend in these MgO–C grades lowers the dependency on temperature. The curve is shifted to higher N-values (Fig. 6). The coefficient of determination is good.

6.2 Bricks within the system SiO_2–Al_2O_3

For high SiO_2 contents (silica, fireclay) and amounts of vitreous phases the thermal conductivity rises with temperature. Bricks with high contents of Al_2O_3 show decreasing values with temperature; but if alumina containing bricks are only tempered and not burned, the thermal conductivity can also slightly rise with temperature (Fig. 7).

Of influence again are contacts within the texture, the growth of the crystals and the porosity.

Datas for the different material groups are taken from Tab. 1. The coefficient of determination is very good.

6.3 Thermal insulation material

Because of their high amount of porosity thermal insulating materials like fibre products and light weight monolithic shows slightly increasing values of thermal conductivity with temperature (like air). Insulating bricks with a high density and high classification temperature can show decreasing

values for the thermal conductivity with temperature (Fig. 8).
Datas for the different material groups are taken from Tab. 1.
The coefficient of determination is in consideration of three different material groups, very good.

6.4 Metallic alloys

Both alloy-groups show the same characteristic in the temperature/thermal conductivity dependence as refractory materials. (Fig. 9).
The n/N tendencies for alloys without Cu, Mg, Al. are within the range of the dependencies for SiC and C.

7 Comparison of calculated with measured values of thermal conductivity

The exponents N and n for three groups of materials depending on two different samples (1, 2) within one material group were calculated from the measured values at 673 K and thus calculates the thermal conductivity at 1073 K and compared with the measured values (Tab. 2).
The deviations are low and are within the range of the repeatability of measurements of thermal conductivity (Fig. 10).

8 Summary

The temperature dependence of the thermal conductivity of refractories is calculated from the linear dependence between the logarithms of the thermal conductivity and the logarithms of the temperature in K. Thus the correlation between thermal conductivity and temperature (300–1400 °C) for refractories can be described with the following equations:

$$\ln \lambda_{eff} = n \cdot \ln\left(\frac{T}{T_0}\right) + N \qquad (1)$$

or

$$\lambda_{eff} = e^{N} \cdot \left(\frac{T}{T_0}\right)^{n} \qquad (2)$$

with

$$e^{N} = \lambda_o \qquad (3)$$

λ_0 is the virtual thermal conductivity with the reference temperature $T_0 = 1$ K.
For refractories, and two groups of metallic alloys (with and without Mg, Al, Cu) there is a linear dependence between the exponents n and N:

$$n = a\,N + b \qquad (6)$$

The mean value for "a" for all refractory material groups with a coefficient of conformity above 0,98 is –0,131. The plain parallel shifting of the strait line is for the covalent bonding of insulating materials approximately slightly negative (b = –0,06), for ionic bonding in oxides b increases approximately up to 0,3 and for metallic bonding for metallic alloys up to 0,7.
For all material groups with a high coefficient of determination of the n/N dependence it is possible to calculate the thermal conductivity/temperature dependence from one single value of the thermal conductivity at one temperature.

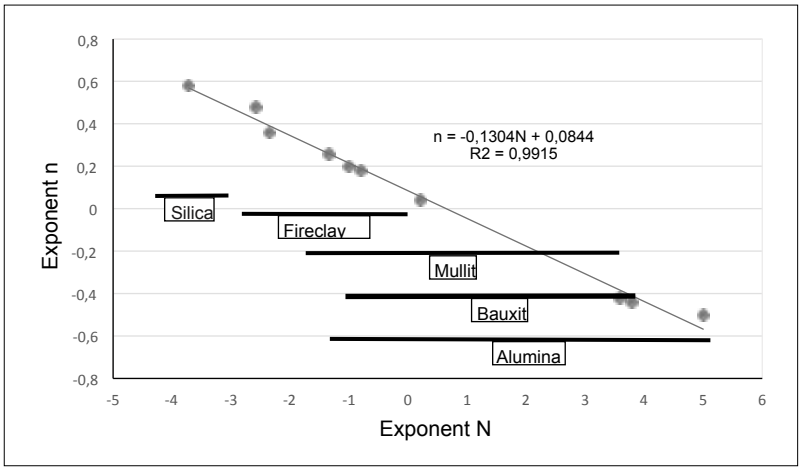

Fig. 7 Range of n/N dependence of bricks within the system SiO_2–Al_2O_3

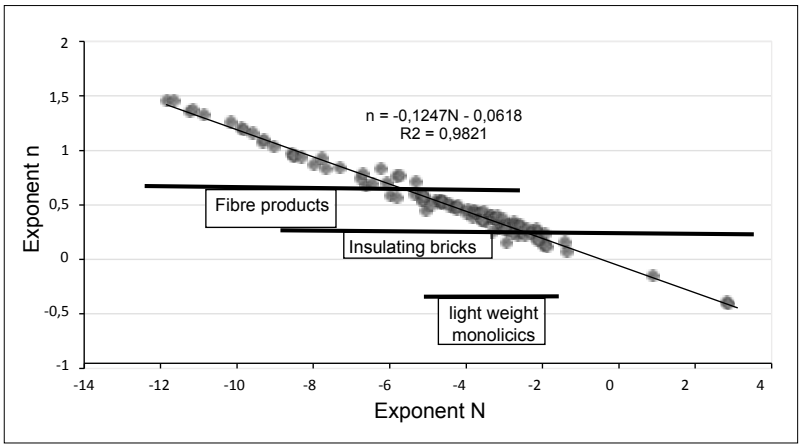

Fig. 8 Range of n/N dependence for different thermal insulating materials

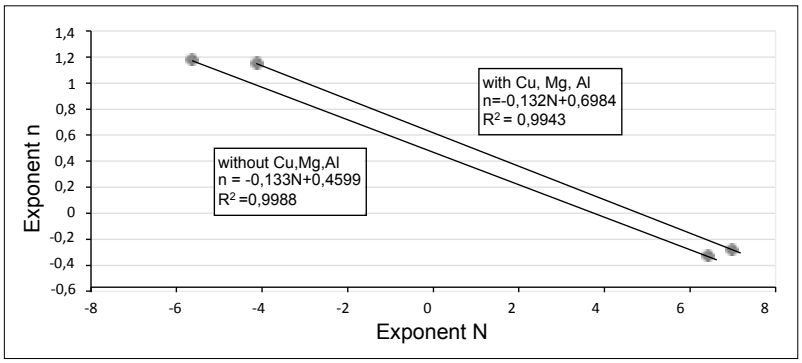

Fig. 9 n/N dependence for two metallic alloy groups

Tab. 2 Calculation examples

Material Group	Sample No.	ln λ	ln T	N	n	λ	λ	Deviation
						Calculated [W/m·K] 1072 K	Measured [W/m·K] 1072 K	[%]
Fibre materials								
n= –0,1247N – 0,0618	1	2,112	6,512	9,095	1,0723	0,199	0,199	0
COD = 0,9968	2	2,226	6,512	–9,640	1,1400	0,185	0,208	–11
MgO materials								
n = 0,1316N + 0,1295	1	2,042	6,512	8,097	–0,9300	4,450	4,680	–5
COD = 0,995	2	1,644	6,512	5,410	–0,5840	3,890	4,100	–5
Mullite materials								
n = –0,1386N + 0,071	1	0,538	6,512	0,419	0,0233	1,787	1,657	7
COD = 0,9979	2	0,604	6,512	0,814	–0,0360	1,759	1,746	1

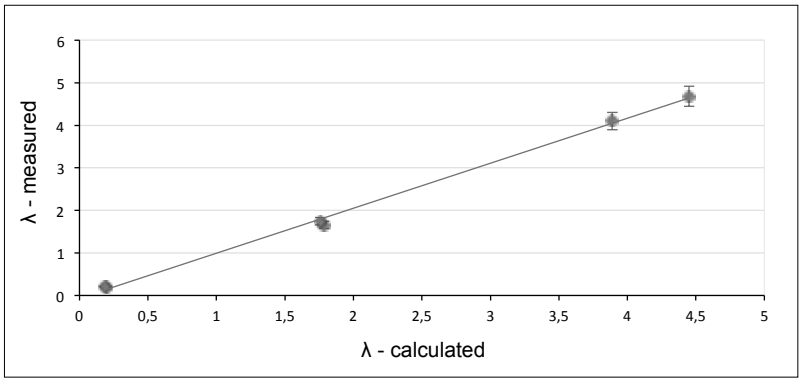

Fig. 10 Deviation between calculated and measured thermal conductivity

Furthermore, with equation (2), the integral mean between two temperatures can be calculated.

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